Department of Mathematics

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Minor 1 (II Semester 2015 - 2016)

Time allowed: 1 hour

Max. Marks: 25

1. (a) Write axiomatic definition of probability.

   (b) Let $\Omega = \{a, b, c, d\}$. Find three different σ-fields $\{\mathcal{F}_n\}$ for $n = 0, 1, 2$ such that $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \mathcal{F}_2$.

   (3 + 3 marks)

2. Consider the function

$$F(x) = \begin{cases} 0, & x \leq 0 \\ c\sin^{-1}(\sqrt{x}), & 0 < x \leq 1 \\ 1, & x > 1 \end{cases}$$

   Find constant c so that $F(x)$ represents the cumulative distribution function of a random variable and verify that it is infact a distribution function.

   (2 + 3 marks)

3. Let X be a random variable such that

$$P(X > x) = \begin{cases} q^{[x]}, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

   where $0 < q < 1$ is a constant and $[x]$ is integral part of x. Discuss whether the distribution of X is discrete or continuous or mixed. Determine the pmf/pdf as applicable to this case.

   (4 marks)

4. In the claim office with one employee of a public service enterprise, it is known that the time (in minutes) that the employee takes to take a claim from a client is a random variable which is exponential distribution with mean 15 minutes. If you arrive at 12 noon to the claim office and in that moment there is no queue but the employee is taking a claim from a client, what is the probability that you must wait for less than 5 minutes to talk to the employee? What is the mean spending time (including waiting time and claim time) of you?

   (3 + 2 marks)

5. A point X is chosen at random in the interval $[-2, 1]$. Find the pdf of $Y = X^2$. (5 marks)